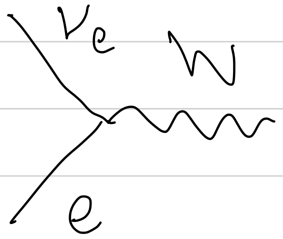
Primer on

Neutrino Oscillations

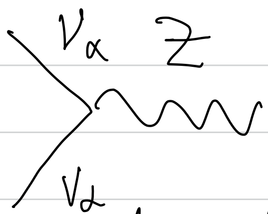
Neutrino Oscillations

In the S.M. neutrinos interact via NC and CC interactions.
In symbols and diagrams:



$\longleftrightarrow$ $\bar{e}_{\alpha L} \gamma^\mu \nu_{\alpha L} W_\mu^-$

and also



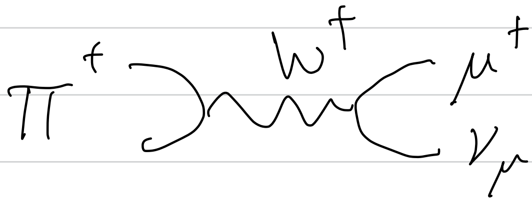
$\longleftrightarrow$ $\bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L} Z_\mu^0$

However, we know that neutrinos have masses. These mass states, ν_i, are not "aligned" with the flavor states above. They are related by a matrix, which we call the PMNS matrix. Namely,

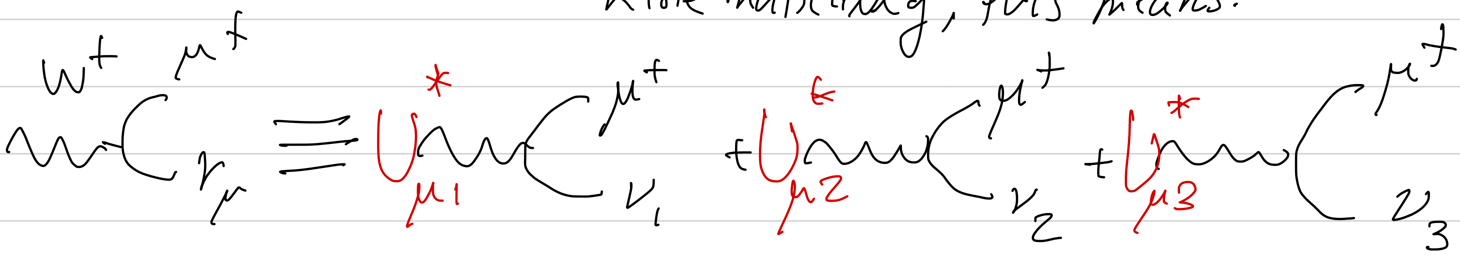
$$\underbrace{|\nu_\alpha\rangle}_{\text{flavor state}} = U^*_{\alpha i} \underbrace{|\nu_i\rangle}_{\text{mass state}}$$

How do we understand neutrino production say from pion decay?

$$\pi^+ \longrightarrow \mu^+ \nu_\mu$$



because the mass of the neutrino "cannot" (so far) be measured kinematically, this means:



We are going to assume neutrinos are a monocromatic plane wave (not true, we will come back to this later)

Then, if the neutrino travels some distance L from production to detection, we can figure out the neutrino final state:

$$|\nu(t)\rangle = e^{-iHt} |\nu(0)\rangle \quad \text{where } t = L.$$

(we are working in god-given units, where $c = \hbar = 1$). Say we start with some flavor α, so:

$$|\nu_{(0)}\rangle = |\nu_\alpha\rangle$$

$\Rightarrow$

$$|\nu_{(t)}\rangle = e^{-iHt} |\nu_\alpha\rangle$$

If neutrinos are propagating in vacuum the Hamiltonian is given by the kinetic energy.

Each mass state will have a well-defined energy E_i, namely

$$\mathcal{H} |\nu_i\rangle = E_i |\nu_i\rangle$$

then the final state is given by:

$$|\nu_{(t)}\rangle = e^{-iHt} \left(\sum_i U^*_{\alpha i} |\nu_i\rangle \right)$$

$$= \sum_i U^*_{\alpha i} e^{-iHt} |\nu_i\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t} |\nu_i\rangle$$

the amplitude of observing a flavor β is given by:

$$\mathcal{A}(\nu_\alpha \to \nu_\beta ; t) = \langle \nu_\beta | \nu(t) \rangle$$

$$\Rightarrow \mathcal{A}_{\alpha\beta}(t) = \langle \nu_\beta | \sum_i U^*_{\alpha i} e^{-iE_i t} | \nu_i \rangle \qquad \left[\text{using } \langle \nu_\beta | = \sum_j U_{j\beta} \langle \nu_j | \right.$$

$$= \sum_{ij} U_{j\beta} U^*_{\alpha i} e^{-iE_i t} \langle \nu_j | \nu_i \rangle$$

$$= \sum_i U_{i\beta} U^*_{\alpha i} e^{-iE_i t}$$

then the probability is given by:

$$P(\nu_\alpha \to \nu_\beta ; t) = \left| \mathcal{A}(\nu_\alpha \to \nu_\beta ; t) \right|^2$$

Two Flavor Case:

Algebra here is not very enlightening; let's do two flavor scenario. In this case:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

then:

$$|\nu_e\rangle = c|\nu_1\rangle + s|\nu_2\rangle$$
$$|\nu_\mu\rangle = -s|\nu_1\rangle + c|\nu_2\rangle$$

$$\mathcal{A}_{\mu e}(t) = \left(\langle \nu_2 | s + c \langle \nu_1 | \right) \left(-e^{-iE_1 t} s |\nu_1\rangle + e^{-iE_2 t} c |\nu_2\rangle \right)$$
$$= sc\, e^{-iE_2 t} - sc\, e^{-iE_1 t} = sc\, e^{-iE_1 t} \left(e^{-i(E_2 - E_1)t} - 1 \right)$$

then the probability:

$$P_{\mu e}(t) = A_{\mu e} A_{\mu e}^{*} = s^2 c^2 \left(e^{-i\Delta E t} - 1\right)\left(e^{+i\Delta E t} - 1\right)$$

$$= s^2 c^2 \left(1 - e^{-i\Delta E t} - e^{+i\Delta E t} + 1\right)$$

$(e^{i\theta} = c\theta + is\theta)$

$$= s^2 c^2 \left(2 - \left(e^{i\Delta E t} + e^{-i\Delta E t}\right)\right)$$

$$= s^2 c^2 \left(2 - 2\cos(\Delta E t)\right)$$

$$= 2 s^2 c^2 \left(1 - \cos(\Delta E t)\right)$$

$$= \frac{\sin^2(2\theta)}{2}\left[2\sin^2\left(\frac{\Delta E t}{2}\right)\right]$$

then:

$$P_{\mu e}(t) = \sin^2(2\theta)\sin^2\left(\frac{\Delta E t}{2}\right)$$

What is $\Delta E = ??$

$$E = \sqrt{p^2 + m^2} \simeq p + \frac{m^2}{2p} \qquad (|p| >> m)$$

Assuming that the neutrinos have same momenta;

$$\Delta E = \frac{\Delta m_{21}^2}{2E} \quad ; E \simeq |p|$$

then:

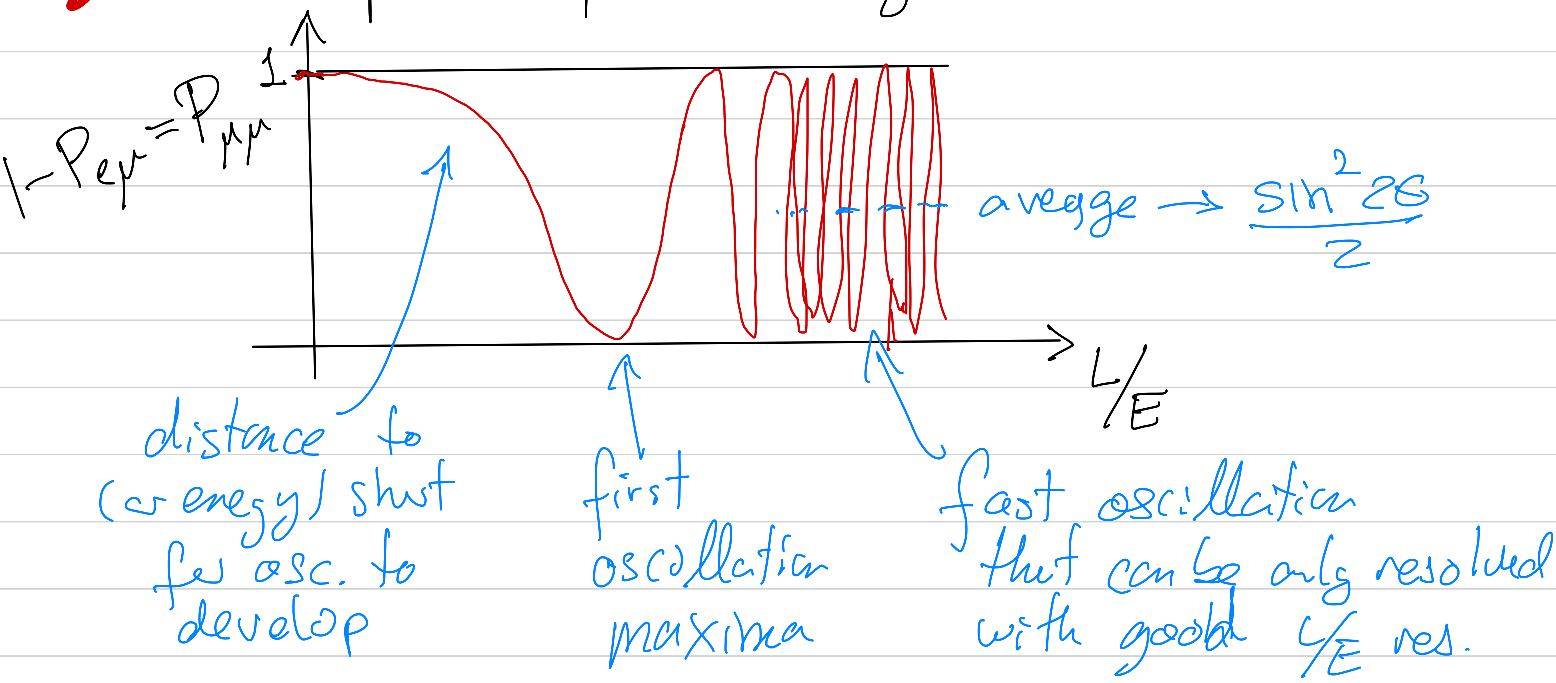
$$P_{\mu e}(t) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) \quad ; L = t.$$

Important Remarks on Neutrino Oscillations

1) The amplitude of neutrino oscillations is set by nature and is given by U. [can be modify in matter...]

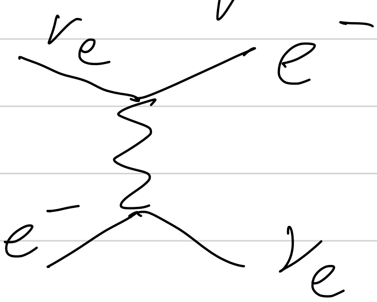
2) Neutrino oscillations (in vacuum) are periodic in L/E.

3) Three important experimental regimes:



4) Neutrinos are natural interferometers; i.e., they are sensitive to the material they travel through if the material interacts with neutrinos.

Matter effects:



$$\Longrightarrow \quad \mathcal{H} = U^\dagger \frac{m^2}{2p} U + \begin{pmatrix} \text{potential} \\ \text{source} \\ \text{by matter} \end{pmatrix}$$

$$\sim G_F \begin{pmatrix} n_e & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

Oscillation Scales For HE neutrinos:

There are two Δm^2 :

$$\Delta m^2_{21} \sim 7.4 \times 10^{-5} \, eV^2$$

$$\Delta m^2_{31} \sim (\pm) \, 2.51 \times 10^{-3} \, eV^2$$

which give you two oscillation frequencies. One can compute the oscillation length (distance to first oscillation maxima) by:

$$l_{osc} = \frac{4\pi E}{\Delta m^2} = 2.48 \, km \left(\frac{E}{GeV}\right)\left(\frac{eV^2}{\Delta m^2}\right)$$

picking the largest Δm^2:

$$l_{osc}^{10 GeV} \approx 10\,000 \, km \quad \approx \text{comparable to Earth diameter}$$

$$l_{osc}^{1 TeV} = 10^6 \, km$$

$$1 \, AU \approx 1.5 \times 10^8 \, km$$

$$l_{osc}^{1 PeV} = 10^9 \, km$$

this implies that for neutrinos from other galaxies we are in the regime of many oscillation lengths.

In the avg. out regime the osc. prob. is given by:

$$P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

$\therefore$ we do not expect to see oscillation features in HE νs.