

Particle acceleration theory II The radiation connection

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Outline

- **Some Alternative Acceleration Mechanisms**
- A unified picture of non-thermal electron emission
- Single zone models
- Examples





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Alternative Acceleration Schemes

- **Magnetic Reconnection**
- **Shear Flows**
- **Relativistic Shocks**
- **Converter Mechanisms**





Magnetic Reconnection

Originally suggested by Giovanelli (1947) and Dungey (1953).

Developed by Sweet, Parker, Furth & Petschek in 50s and 60s.

Plays major role in solar physics

- magnetic flares
- solar storms
- coronal / solar wind heating?

Important in pulsars, magnetars & possibly GRBs/AGN jets



Particles that reach $r_g > \delta$ can tap into E_z

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Schindler & Hornig 01



Sweet - Parker picture

Particles drift into diffusion region

$$v_{in} = \frac{E \times B}{B^2} \sim \frac{E_z}{B}$$

here $E_z = \eta j \sim \eta \frac{B_1}{\delta}$

• Mass conservation implies $\Delta v_{in} = \delta v_{out}$ and energy conservation $B_1/8\pi = \rho v_{out}^2/2$ Hence $v_{in}/v_{out} = v_{in}/v_{Alfven} = \delta/\Delta \ll 1$

We can combine the above to show $v_{in} \approx v_{Alfven} / \sqrt{R_m}$

W

- Simulations show that "fast" reconnection is possible.
- Jury still out on shape of non-thermal spectrum





Acceleration in Shear Flows



Assuming energy conserved in local frame $\frac{\Delta E}{E} = \beta_j \frac{\cos \theta_2 - \cos \theta_1}{1 - \beta \cos \theta_2}$

If particle exits with $\cos \theta_2 \approx 1$, fractional energy change $\Delta E/E \sim \Gamma_i^2$, though such large energy gains are rare

Can produce very hard power-laws if particle do not scatter in jet sheath. Otherwise, spectra are softer (See Rieger & Duffy 07, Wang et al 21 for details)



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e.g. NGC315 Worrall et al 2007







Acceleration at relativistic shocks





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VHE/UHE *γ***-ray observations of**

Gamma-Ray Bursts (to > 10 TeV) And Pulsar Wind Nebulae (to PeV!)

Suggest extremely efficient accelerators.

Acceleration at relativistic shocks remains a possible mechanism for UHECR production





Relativistic vs non-relativistic shocks

A shock moving with speed close to the speed of light In hydrodynamic limit, flow comes in with $u_{-} \approx c$, exits at $u_{+} \approx c/3$

Perpendicular component of magnetic field is Lorentz boosted in shock rest frame.

Particles in upstream must outrun the shock $|v\mu$ Since $v \approx c$, it must be that $1 - \theta^2/2 > u_{sh}/c \rightarrow d^{sh}/c$



Numerical simulations show acceleration occurs from 1st principles, if magnetic field is *weak* (not the case for pulsars)



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$$\begin{aligned} u \mid &> u_{sh} \\ \theta < 1/\Gamma_{sh} \end{aligned}$$

Spectrum is typically $dN/dE \sim E^{-2.2}$ Matches well many observations



Converter Mechanism



E.g. $p + \gamma$

a neutron can outrun the shock without deflection. If it converts back to proton in a subsequent collision, a large energy gain is possible.

Spectrum is thought to be hard

Derishev, Aharonian, et al. 2003



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A particle can undergo extreme Fermi cycles $\Delta \gamma / \gamma \sim \Gamma^2 \Delta \theta^2$, if the bulk Lorentz factor is large, and the optical depth to inelastic collisions is order unity.

$$\rightarrow \begin{cases} p + \pi^0 \\ n + \pi^+ \end{cases}$$





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A unified approach to electron emission

Consider an electron sitting in path of an electromagnetic plane wave $A = A_0 \cos(kx - \omega t)\hat{y}$



The electron accelerates in the electric field as $\ddot{y} = -\frac{e}{-E}$,

But accelerating charges radiate like a Hertzian Dipole $L = \frac{2}{2} \frac{(e\ddot{y})^2}{2} = \frac{8\pi}{2} \left(-\frac{1}{2} \frac{(e\ddot{y})^2}{2} \right)$ $L = \sigma_T \ u_{\text{light}} \ c$



$$(B = \nabla \times A, E = -\frac{1}{c}\frac{\partial A}{\partial t}, B_0 = E_0 = (\omega/c)A_0)$$





A unified approach to electron emission

Nothing on previous slide relied on the plane wave assumption, only that the Electric field "shook" the electron.

Thus we can generalise the power to radiated to $L = 2\sigma_T \langle u_{\text{elec}} \rangle c$

We seek to generalise this to relativistic particles. In the following slides we take the ultra-relativistic limit $\gamma \gg 1$ and consider in turn synchrotron and inverse Compton emission





- 1. The electric field in the electron's rest frame i
- 2. From this we find boosted power : $L' = 2\gamma^2 \sigma_2$
- 3. Now if in lab frame, E=0, we have $L' = 2\gamma^2 \beta_{\perp}^2$
- 4. Finally emitted power $L' = \frac{dE'}{dt'} = \frac{dE/\gamma}{dt/\gamma} = \frac{dE}{dt} = L$, i.e. radiated power is a Lorentz inv.

Thus we have derived the synchrotron power formula $L = 2\gamma^2 \beta_{\perp}^2 \sigma_T c u_{mag}$

$$L = 2\sigma_T \left< u_{\rm elec} \right>$$

$$\int_{T} e^{AS} \frac{E' = \gamma (E + \beta \times B)}{(E + \beta \times B)^{2}}$$

$$\int_{T} e^{C} \frac{(E + \beta \times B)^{2}}{8\pi}$$

$$\int_{T} e^{AS} \frac{B^{2}}{8\pi} = 2\gamma^{2} \beta_{\perp}^{2} \sigma_{T} c u_{\text{mag}}$$



$$< L > = \frac{4}{3}\gamma^2\beta^2\sigma_T c u_{\rm ma}$$









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JS starts observing: $t_1 = 0 + (R - v\Delta t)/c$ **JS stops observing:** $t_2 = 2\Delta t + (r + v\Delta t)/c$ JS sees pulse $t_2 - t_1 = 2\Delta t (1 - v/c) \approx \Delta t/\gamma^2$ Characteristic frequency $1/\Delta t_{obs} = \gamma^3 \omega_{\varphi}$

Fun facts:

Synchrotron is like a lighthouse effect with crit

Detailed derivation of single particle power L_{ν}

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ical observed frequency
$$\nu_c = \frac{3}{4\pi} \gamma^3 \omega_g = \frac{3}{4\pi} \gamma^2 \frac{eB_\perp}{m_e c}$$
.
$$= \frac{dE}{dt \ d\nu} = \sqrt{3} \frac{e^2}{c} \omega_g F\left(\frac{\nu}{\nu_c}\right) \text{ where } F(x) = x \int_x^\infty K_{5/3}(\eta) dx$$

Spectrum is broad, peaks at $\nu \approx 0.29 \nu_c$ Blue line shows exact solution **Orange line shows** $F_{approx}(x) = 1.85x^{1/3}e^{-x}$

$$F(x) \sim \begin{cases} x^{1/3} & x \ll 1\\ \sqrt{x}e^{-x} & x \gg 1 \end{cases}$$

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If electron spectrum is smooth, luminosity is $\frac{dE}{d\nu dt d\Omega} = \int \frac{dN}{dE_e d\Omega} L_\nu dE_e \approx \frac{1}{4\pi} \int \frac{dN}{dE_e} L_\nu dE_e$

Simple approach: let $L_{\nu} = \langle L_{svn} \rangle \delta(\nu - 0.29\nu_c)$

Consider a power-law of electrons $dN/dE \propto E^{-s}$ $\langle L_{svn} \rangle \propto E^2 B^2$, $0.29\nu_c = a_0 E^2 B$

$$(E^2 B^2) E^{-s} (EB)^{-1} \Big|_{E=\sqrt{\nu/a_0 B}} \propto \nu^{-(s-1)/2} B^{(s+1)/2}$$

Fun facts continued: Synchrotron photons peak at $\frac{h\nu_c}{m_c c^2} = 0.29 \frac{3}{4\pi} \gamma$

Energy radiated as particle sweeps by line of

"Number" of photons emitted is $N_{\rm ph} = \frac{L\Delta t}{h\nu} \approx$

$$\gamma^{2} \frac{heB_{\perp}}{m_{e}^{2}c^{3}} = 0.44\gamma^{2} \frac{B}{B_{\text{crit}}} \quad \text{where } B_{\text{crit}} = \frac{m_{e}^{2}c^{3}}{\hbar e}$$
sight $L\Delta t = \frac{4}{3}c\sigma_{T}\gamma^{2}u_{mag} \times \frac{1}{\gamma\omega_{g}} \approx \gamma^{2}\alpha_{f}\frac{B}{B_{\text{crit}}}m_{e}c^{2}$

$$\alpha_f$$
 $(\alpha_f = \frac{e^2}{\hbar c} \approx \frac{1}{137}$ is fine-structure consta

The synchrotron burn-off limit

Recall also the gyro period $P = \frac{2\pi}{\omega_g} = \frac{2\pi\gamma m_e c}{eB} \propto \frac{\gamma}{B}$

These two times are equal when $\gamma^2 \frac{B}{B_{\text{crit}}} = \alpha_f^{-1}$

But we already saw $\frac{h\nu_c}{m_c c^2} = \frac{3}{2}\gamma^2 \frac{B}{B_{crit}} \approx \alpha_f^{-1}$ i.e. there is a theoretical upper limit for synchrotron photon energies of a few 100 MeV (Though there are ways around this....)

- Consider a relativistic particle in an *isotropic* photon field
- 2. The energy density transforms as the 00th element of the energy-momentum tensor : $T^{\mu\nu} = p^{\mu}p^{\nu}u_{elec}$

$$u'_{\rm ph} = \gamma^2 (1 - \beta \cos \theta)^2 u_{\rm ph} \rightarrow \langle u'_{\rm ph} \rangle = \gamma^2 \left(1 + \frac{\beta}{2} \right)^2 u_{\rm ph}$$

3. radiated power is again a Lorentz inv.

Thus we have derived the IC power formula L_{IC} Principia Programme in Multi Messenger Astrophysics - São Paulo 2023

 $L = 2\sigma_T \langle u_{\text{elec}} \rangle c$

Note k^{μ} is a 4-vector $\nu' = \gamma \nu (1 - \beta \cos \theta)$ $\nu'' = \gamma^2 (1 + \beta \cos \theta) (1 - \beta \cos \phi) \nu$ $\frac{1}{4\gamma^2} < \frac{h\nu''}{h\nu} < 4\gamma^2 \text{ (careful!)}$

$$\begin{pmatrix} 2 \\ - \end{pmatrix} u_{\rm ph}$$

$$c = \sigma_T \langle u'_{\rm ph} \rangle \ c = \frac{4}{3} \gamma^2 \sigma_T c u_{\rm mag}$$

A unified approach to electron emission II - Inverse-Compton emission **Consider a monochromatic photon field with energy** $\epsilon_0 = h\nu_0$, and energy density $u_{ph} = n_{ph}\epsilon_0$

We saw the scattered photon has energy $\epsilon_1 = \gamma^2 (1 + \beta \cos \theta) (1 - \beta \cos \phi) \epsilon_0$ and since θ and ϕ are likely uncorrelated, average up scattered photon has energy $\gamma^2 \epsilon_0$.

e.g. consider a CMB photon $\epsilon_0 \approx 6.6 \times 10^{-4} eV$, up-scattered by a TeV electron $\gamma = 2 \times 10^{6}$ This gives $\epsilon_1 = 2.6$ GeV.

The same electrons in a 10 μ G magnetic field would emit 0.2 eV synchrotron photons

If electron spectrum is smooth, luminosity is $\frac{dE}{d\nu dt d\Omega} \approx \frac{1}{4\pi} \int \frac{dN}{dE_e} L_\nu \ dE_e \ ,$ though L_{ν} is a bit more involved

- - Simple approach: let $L_{\nu} = \langle L_{IC} \rangle \delta(\nu \gamma^2 \nu_0)$ Repeating steps from before, we find

$$\frac{dE}{d\nu dt d\Omega} \approx \frac{\langle L_{IC} \rangle}{4\pi} \frac{dN}{dE_e} \left| \frac{dE_e}{d\nu} \right|_{\nu = \gamma^2 \epsilon_0} \propto \nu^{-(s-1)/2}$$

A unified approach to electron emission

Note, we could play the same game with other process, eg. pp interactions

$$L_{pp} = \dot{E}_{pp}\delta(E_{\gamma} - E^*) \text{ where } E^* \approx 0.1E_p \text{ and } \dot{E}_{pp} = \frac{E_p}{t_{pp}} \text{ where } t_{pp} = 10^7 n_{gas}^{-1} \text{ years}$$

$$E_{\gamma}^{2} \frac{dN}{dEdtd\Omega} \approx E_{\gamma} \frac{\langle L_{pp} \rangle}{4\pi} \frac{dN}{dE_{p}} \frac{dE_{p}}{dE_{\gamma}} \bigg| \approx E_{\gamma}^{2-s}$$

However, a TeV electron up scattering a UV photon (at say $\epsilon_0 = 10 \ eV$) gives $\epsilon_1 = 40$ TeV.

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e.g. a CMB photon $\epsilon_0 \approx 6.6 \times 10^{-4} eV$, up-scattered by a TeV electron $\gamma = 2 \times 10^6$ gives $\epsilon_1 = 2.6$ GeV.

We neglected the quantum recoil of the electron. The Klein Nishina effect.

The corrections effectively amount to a reduction in the cross-section $\sigma_{KN} = \sigma_T f(4\gamma \epsilon_0/mc^2)$

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A more precise calculation shows $\frac{\epsilon_1}{\gamma mc^2} < \frac{4\gamma \epsilon_0/mc^2}{1+4\gamma \epsilon_0/mc^2}$ i.e. our results hold only if $4\gamma^2 \epsilon_0 \ll \gamma mc^2$

Exact (Jones 1968) $(1 + x)^{-1.5}$ (Moderski et al. 2005) $(1+x)^{4/3}(1+x/40)^{1/2}$

The corrections effectively amount to a reduction in the cross-section $\sigma_{KN} = \sigma_T f_{KN} (4\gamma \epsilon_0 / mc^2)$

Klein Nishina suppression does 2 things

a) suppresses emission as $\epsilon_1 \rightarrow \gamma mc^2$ **b)** Suppresses cooling for $\gamma > \epsilon_0/mc^2$

In the following slides I use simple cartoons. Serious astrophysicist can use the open source code GAMERA to determine cooling times + spectra

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Consider some region with gas, magnetic and photon fields.

Acceleration occurs "somewhere" and injects energetic particles continuously into this zone, at some rate Q_{ini}

Particles subsequently cool via radiation (We consider for simplicity only IC + synchrotron) $\dot{E}_{cool} = -\frac{4}{3}\sigma_T c \gamma^2 u_{mag} \left[1 + \frac{u_{ph} f_{KN}(x)}{u_{mag}} \right]$ We can construct a time dependent equation for the evolution of the particle distribution $\frac{\partial N}{\partial t} + \frac{\partial}{\partial E} (\dot{E}_{cool}N) = -\frac{N}{t_{esc}} + Q_{inj}(E)$ t_{esc}

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial E} (\dot{E}_{cool}N) = -\frac{N}{t_{esc}} + Q_{inj}(E)$$

If we neglect escape, we can write

$$\frac{\partial N}{\partial t} - \frac{\partial}{\partial E} (bE^2N) = Q_{inj}(E) \text{ where } \dot{E}_{cool} = -b$$

This implies a cooling time $t_{cool} = 1/bE$, such that for $E \ll 1/bt$, losses are irrelevant, while for $E \gg 1/bt$ the system reaches equilibrium. Hence we simply solve 2 approximate equations

$$\frac{\partial N}{\partial t} = Q_{inj}(E) \quad \text{(if } E \ll 1/bt) \qquad N = \int_0^t Q_{inj} \, dt = Q_{inj} \, t$$
$$-\frac{\partial}{\partial E} (bE^2 N) = Q_{inj}(E) \quad \text{(if } E \gg 1/bt) \qquad N = \frac{1}{|\dot{E}_{cool}|} \int_E^{E_{max}} Q_{inj} dE'$$

$$N = Q_{\text{inj}} t \qquad E < 1/bt$$
$$N = \frac{1}{|\dot{E}_{cool}|} \int_{E}^{E_{max}} Q_{\text{inj}} dE' \qquad E > 1/bt$$

Now lets see what happens if we inject $Q_{inj} = Q_0 E^{-s}$ $E_0 < E < E_{max}$

$$N = \frac{Q_0 E^{-s} t}{Q_0} E^{-(s+1)} \qquad E < 1/bt$$
$$N = \frac{Q_0}{b(s-1)} E^{-(s+1)} \qquad E > 1/bt$$

We produce a break in the spectrum

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Cooling Zone

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Photon Spectrum ($u_{mag} > u_{ph}$ **) No KN**

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Photon Spectrum ($u_{mag} > u_{ph}$ **) KN**

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rum
$$(u_{mag} < u_{ph})$$
 KN
pectrum
 $Iog E$
 $I = \frac{1}{|\dot{E}_{cool}|} \int_{E}^{E_{max}} Q_{inj} dE'$
Cooling ti
 $I = \frac{1}{|\dot{E}_{cool}|} \int_{E}^{E_{max}} Q_{inj} dE'$

 10^{2}

 10^{1} +

 10^{-0}

Resulting photon spectrum

 10^1 10^2 10^3

E [TeV]

=

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Eta Car - A Colliding Wind Binary

A time dependent laboratory

Ohm et al. 2015

An astroparticle physics laboratory

<u>η Car A:</u>

- Asymptotic wind speed 500 km/s
- Surface field ~ $100 \ \mu G$
- Max NRG limited by pp cooling

<u>η Car B:</u>

- Asymptotic wind speed 3,000 km/s
- Surface field (probably) $\ll 100 \ \mu G$
- Maximum energy limited by system size

lons ~10% of wind power Electrons ~ 0.3%

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White et al. 21

Pulsars and their Nebulae - the Crab

Powered by spinning NS

The pulsar emits a magnetised **pair** wind.

The mass loading parameter

 $\mu \equiv \frac{L_{\text{Tot}}}{\dot{M}c^2} = \Gamma_w(\sigma_w + 1) \qquad \text{(Michel 1969)}$

is a conserved quantity in absence of losses/pair creation

Here σ_{w} is the magnetisation (ratio of Poynting to enthalpy density flux)

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Kennel & Coroniti '84

For the spinning NS in the Crab,

 $\mu_{LC} \approx \Gamma_{LC} \sigma_{LC} \approx \frac{r_{LC}}{r_o} \frac{n_{GJ}}{n} \approx 10^{6-7}$ (Kirk & Lyubarsky 01)

Take wind at light cylinder to be trans-Alfvenic $\Gamma_{LC} \approx \sqrt{\sigma_{LC}} \sim 10^2$

If σ decreases, Γ_{w} must increase. No time to reconnect field. Shock is likely highly relativistic and strongly magnetised

Acceleration at the Crab wind termination shock?

Naively, one might think no.

PIC simulation of a highly magnetised electron-positron shock.

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Coroniti '90

Cerutti et al. '20

Giacinti & Kirk '18

Acceleration in the equatorial zone

Using as Q_{inj} in a single zone model We can get a reasonable fit to both hard X-rays and LHAASO data points

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MC simulations by Giacinti et al, in prep

Keyingredients

Pulsars are powered by their spin down luminosity. This is carried mostly by a Poynting flux $4\pi R^2 u_{mag}c = \eta L_{SD}$ (assuming spherical symmetry.)

We can re-arrange to give
$$BR = \sqrt{\frac{2\eta L_{SD}}{c}}$$
 or using

$$\varepsilon_{\rm max} = e\beta BR \approx 2.5 \left(\frac{\eta L_{\rm SD}}{10^{36} \text{ erg s}^{-1}}\right)^{1/2} \text{ PeV} \text{ For t}$$

If we take into account losses recall
$$\gamma_{\text{max}}^2 \frac{B}{B_{\text{crit}}} = \alpha_f^{-1}$$

 $\varepsilon_{\text{max}} = 10 \left(\frac{r_{\text{sh}}}{0.1pc}\right) \left(\frac{\eta L_{\text{SD}}}{10^{36} \text{ erg s}^{-1}}\right)^{-1/4} \text{PeV}$

(that the Crab produces PeV photons suggests very high efficiency)

While the Crab is very powerful, having a relativistic shock and an oblique pulsar are not unique. Principia Programme in Multi Messenger Astrophysics - São Paulo 2023

the Hillas limit

the crab $L_{\rm SD} \sim 10^{38} \, {\rm erg \ s^{-1}}$

which gives

UHE gamma-ray sources

UHE Crab nebula is a point like source But HAWC + LHAASO have revealed many extended sources in the 100 TeV sky.

LHAASO collab. Arxiv.

Can these be pulsars? Resolution makes this difficult. Clearly requires hard spectra. But as we saw, hard spectra can be produced if $u_{ph} > u_B$

HAWC Collaboration PRL '20

Cooling in photon dominated volumes

 $\varepsilon_{\max} \propto \begin{cases} BR_{acc} & \text{Hillas limit} \\ B^{-1/2} & Cooling limited \end{cases}$

Field should not be too weak due to size limitations $U_{B,min} \approx 4R_{\rm pc}^{-4/3} \,\,{\rm eV}\,\,{\rm cm}^{-3}$

This may make large u_{ph}/u_B a challenge near acceleration site.

Diffusion away from accelerator minimises synchrotron losses.

Kennel & Coroniti '84

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A reasonable choice of parameters suggests that these pulsars are all capable of producing hard IC spectra.

We eagerly anticipate the data from LHAASO, and no doubt surprises

Many things not covered

- Gamma ray bursts prompt and afterglow emission
- **Blazars and other AGN**
- Magnetars, FRBs, Novae
- **Crab** flares
- **Tidal disruption events**
- **Dark sources**
- **UHECR** production
- Hadronic cascade models, pair-production, turbulent reconnection
- Non-thermal dark matter signatures •
- Etc. etc. the non-thermal universe is vast

To summarise

- There are many non-thermal processes at play in astrophysics
- Theorists always require new observations to test their models, and develop new theories
- Simple single zone models are a good place to start, but a bad place to stop
- It is important to understand limitations and subtleties of radiative processes
- Shock acceleration is still the best developed theory (hence the biased talks), but can not explain all observations

